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Abstract: Recent research has highlighted the necessity of developing routing protocols for mobile ad hoc networks where end-to-end multi-hop paths may not exist and communication routes may only be available through time and mobility. Depending on the context, these networks are commonly referred as Intermittently Connected Mobile Networks (ICNs) or Delay/Disruption Tolerant Networks (DTNs).

Conversely, little is known about the inherent properties of such networks, and consequently, performance evaluations are often limited to comparative simulations (using mobility models or actual traces).

The goal of this paper is to increase our understanding of possible performances of DTNs. After introducing our formal model, we use analytical tools to derive theoretical upper-bounds of the information propagation speed in wireless mobile networks. We also present some numerical simulations to illustrate the accuracy of the bounds in numerous scenarios.

Key-words: MANET, intermittently connected networks, information theory, information propagation, speed, performances.

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Vitesse de Propagation de l'Information dans les Réseaux Mobiles Tolérants aux Délais

Résumé : Des travaux récents ont montré la nécessité de développer des protocoles de routage pour les réseaux mobiles ad hoc où un chemin multi-sauts entre toute paire de sommet n'est pas guarantie, et où donc les communications ne peuvent se faire qu'en combinant le temps et la mobilité. Suivant le contexte, ces réseaux sont appelés "réseaux mobiles à connexion intermittente (ICNs)" ou "Réseaux Mobiles Tolérants aux Délais et aux Dérangements (DTNs)".

En contre partie, il y a peu de connaissance sur les propriétés intrinsèques de ces réseaux, et du coup, les évaluations de performance sont limitées à des comparaisons de simulations (utilisant des modèles de mobilité spécifiques et des traces réelles).

Le but de ce rapport de recherche est d'accroitre notre connaissance des performances possibles dans ces réseaux DTNs. Après avoir défini notre modèle d'étude, nous utilisons des outils analytiques pour obtenir des bornes supérieures théoriques de la vitesse de propagation de l'information dans les réseaux mobiles ad hoc. Nous présentons également des simulations numériques pour illustrer l'exactitude de nos bornes dans de nombreuses configurations.

Mots-clés : MANET, réseaux à connexion intermittente, théorie de l'information, propagation de l'information, vitesse, performances.

1 Introduction

Our objective is to evaluate the maximum speed at which a piece of information can propagate in a mobile wireless network. If the network is connected (i.e., an end-to-end multi-hop path exists) this speed is rather high and can be considered infinite compared to the mobility of the nodes: a piece of information is a packet (of small size) which can be transmitted almost instantaneously between two nodes in range.

A major difficulty occurs when the mobile network is temporary disconnected. In this case the information propagation is stalled as long as the node mobility does not allow the information to jump to different connected components of the network. The packet is either transmitted or carried by a node (requiring a *store-carry-and-forward* routing model). Thus, a "path" is an alternation of packet transmission and packet carriage that connects a source to a destination, and is better referred (from now on) as a **journey**.

Depending on the context, these networks are commonly referred as Intermittently Connected Mobile Networks (ICNs) or Mobile and Delay/Disruption Tolerant Networks (DTNs). Numerous efforts have been dedicated to the design of efficient routing protocols (see [13] for a survey of most results).

Unfortunately, performance evaluations are often limited to comparative simulations, using concrete traces (*e.g.,* [11, 14]) or specific mobility models [3], as a complete understanding of what one can expect for optimal performance (*e.g.,* through theoretical bounds) is still missing.

Informally, our aim is to find the shortest journey (in time) that connects a source to a destination. (Note that formally, we should denote this as the "earliest" or "foremost" journey as the smallest time required between a particular source and a particular destination will vary with the time of the request. However, as we are in fact interested in the overall propagation speed, between any source and any destination, we will denote it as the shortest journey without lack of generality.) It is worth noting that the shortest journey in time from a source to its destination can be calculated in polynomial time when the changes in the network topology can be predicted in advance (*e.g.,* [2, 4, 7]).

Without predictive knowledge, we can still achieve the fastest possible information propagation, by using an algorithm that contains all possible "shortest" journeys: the full broadcast. We call the information, the *beacon*. Every time a new node is in range of a node which carries a copy of the beacon, the latter node transmits another copy of the beacon to the new node. Transmission is done indifferently by broadcast or unicast, this does not change the details of our model.

We do not consider here the fact that the high rate of transmission will affect the range of transmission (*e.g.,* Gupta and Kumar [6]) and that packet loss will occur: our model is intended to unfold all journeys from a source to a destination for which a window of opportunity for transmission may be possible.

1.1 Mobile network model

We consider an infinite two-dimensional network with uniform node distribution and with constant density ν. We assume that each node follows an independent walk of speed v. The node keeps a uniform speed but changes direction at Poisson rate τ.

When $\tau \to \infty$ we are on the Brownian limit, when $\tau \to 0$ we are on a random way point-like model. The motion direction angles are uniform distributed between 0 and 2π.

Initially, we also adopt the unit-disk model: two nodes at distance smaller than one can exchange the beacon. The average number of neighbors per node is therefore $\pi\nu$.

In [12], Xue and Kumar have shown that if the average number of neighbors is smaller than $0.074 \log N$, N being the total number of nodes in the network then the network is surely disconnected.

The network in our model being infinite it is surely disconnected. However the accumulated size of neighbors increasing with time, there exists almost surely a journey between a source and a destination if one waits long enough (*i.e.,* the accumulated topology becomes connected over time [9]).

1.2 Main results

Our main result is the evaluation of an upperbound of the information propagation speed. In fact the concept of propagation speed is probabilistic. Let consider a mobile at coordinate $\mathbf{z} = (x, y)$ at time $t = 0$, let $q(\mathbf{z}, t)$ denotes the probability that the mobile receives the beacon before time t. A scalar $\sigma_0 > 0$ is an upper bound of the propagation speed, if for all $\sigma > \sigma_0$ $\lim q(\mathbf{z}, \frac{|\mathbf{z}|}{\sigma}) = 0$ when $|\mathbf{z}| \to \infty$. For example if $q(\mathbf{z}, t) < \exp(-a|\mathbf{z}| + bt + c)$ then quantity $\frac{b}{a}$ is a propagation speed upperbound.

Theorem 1 *An upper bound of the information propagation speed in a network (where node radio range is R, node density is ν, node speed is v, node direction change rate is τ) is the smallest ratio $\frac{\theta}{\rho}$ attained by the elements (ρ, θ) of the set $\mathcal{K}$ made of the non negative tuples (ρ, θ) that are root of*

$$\sqrt{(\tau+\theta)^2 - \rho^2 v^2} - \tau - \frac{\frac{8vR}{\pi}\nu I_0(\rho R)}{1 - \pi R^2 \nu \Psi(\rho R)}$$

where functions $I_0()$ and $\Psi()$ can be expressed in terms of modified Bessel functions *(see [1]) and are defined respectively by*

$$I_0(x) = \sum_{k \geq 0} (\frac{x}{2})^{2k} \frac{1}{(k!)^2}$$

and

$$\Psi(x) = \frac{2}{x} I_1(x) = \sum_{k \geq 0} (\frac{x}{2})^{2k} \frac{1}{(k+1)!k!}.$$

Remark Quantity ρ is expressed as inverse of distance and quantity θ is expressed as an inverse of time, therefore the ratio $\frac{\theta}{\rho}$ is the dimension of a speed. In the remaining of the paper, and *w.l.o.g.*, we will assume that $R = 1$.

Since quantities $I_0(x)$ and $\Psi(x)$ are both greater than 1, the previous expression has meaning when $\nu < \frac{1}{\pi}$. When $\nu \geq \frac{1}{\pi}$, our model indicates an infinite propagation speed. Therefore our model is interesting when ν is small. Similarly, when $v = 0$ and $\nu < \frac{1}{\pi}$, the propagation speed is zero.

Corollary 1 *Let $v > 0$ and $\tau > 0$, when $\nu \to 0$, the propagation speed is asymptotically equivalent to $4v\sqrt{\frac{\nu v R}{\tau \pi}}$.*

It is important to notice that the speed diminishes with the square root of the density ν.

However this estimate does not hold in the case $\tau = 0$ that we will fully depict as the random waypoint model limit later in the paper.

Corollary 2 *In the random waypoint limit, that is $\tau = 0$, the propagation speed upper bound is $(1 + O(\nu^2))v$.*

Remark It turns out that the propagation speed upperbound at the limit is v. It is rather surprising because we would expect that the propagation speed tends to zero when $\nu \to 0$. In fact the corollary is not only correct but also accurate as we will confirm by simulations. The explanation is that we first set ν and then look at the propagation speed when the mobile nodes are located at location infinitely far from the beaconing source. This is different than considering first a node at a remote location $\mathbf{z}$ from the source and then let $\nu \to 0$. In fact we have $q(\mathbf{z}, t) < \exp(-a|\mathbf{z}| + bt + c)$ and $c \to \infty$ when $\nu \to 0$ confirming that propagation speed tends to zero when $\nu \to 0$ when $\mathbf{z}$ is fixed, but tends to $(1 + O(\nu^2))v$ when ν is fixed and $|\mathbf{z}| \to \infty$.

A rule of thumb for having v lower-bounding the propagation speed is that as soon as the set of attained nodes is large enough there is likely to be one heading toward $\mathbf{z}$.

1.3 Plan of the paper

We adopt a didactic approach. In Section 2, we will initially study a simpler problem where nodes are only allowed to transmit when they change direction or when they receive the beacon. This is not an upper-bound since we do not allow nodes to transmit when they move (although it is worth noting that it will converge to the real problem when we consider the Brownian limit). We will develop and comment our analytical tools in this simpler approach. In Section 3, we will complicate the approach to make it general (and realistic), by allowing nodes to transmit only when they have a new neighbor or when they receive a new copy of the beacon (*i.e.,* when a local event occurs). In Section 4, we present some numerical simulations to illustrate the accuracy of the bounds in numerous scenarios.

2 Simplified approach

In this section we suppose that nodes can only transmit when they change direction or when they receive the beacon.

2.1 Journey analysis

Our analysis is based on journey segmentation between the source and the destination. Formally, a journey is a space-time trajectory of the beacon between the source and a destination. We assume that time zero is when the source transmits, and we will check at what time t, the beacon is emitted at distance smaller than one to the destination at coordinate $\mathbf{z} = (x, y)$. The beacon can take many journeys in parallel, due to the broadcast nature of radio transmission, and the fact that the beacon stays in the memory of the emitter (and therefore can be emitted several times in the trajectory of a mobile node). In a first approach and in order to simplify we will assume that the destination is fixed (the node does not move). We will later modify the model in order to support the destination motion.

We will consider only simple journeys, *i.e.,* journeys which never return twice through the same node. If a journey arrives to the destination at time t then we can extract a simple journey from this journey which arrives at time t too.

To simplify the presentation we now consider journeys as if they were enumerable objects and therefore can be affected to a probability weight. In the following we will in fact consider a journey as a discrete event in a continuous set and therefore the probability weight should be converted into a probability density.

Let $\mathcal{C}$ be a simple journey. Let $Z(\mathcal{C})$ be the terminal point. Let $T(\mathcal{C})$ be the time at which the journey terminates. Let $p(\mathcal{C})$ be the probability of journey $\mathcal{C}$.

We call $p(\mathbf{z}, t)$ the average number of journeys that arrive at $\mathbf{z}$ before time t:

$$p(\mathbf{z}, t) = \lim_{r\to\infty} \frac{1}{\pi r^2} \sum_{|\mathbf{z}-Z(\mathcal{C})|<r, T(\mathcal{C})<t} p(\mathcal{C}) \ .$$

2.1.1 Journey segmentation

In the following we split the journey into segments $\mathcal{C} = (s_1, s_2, \ldots, s_k)$ such that $p(\mathcal{C}) = p(s_1)p(s_2)\cdots p(s_k)$.

Therefore a journey is made by two kinds of segments:

- Carry segment s_c: the beacon is hold until the next change of direction;
- Emission segment s_e: the beacon is transmitted to a neighbor.

A carry segment s_c is a space-time vector $(tv\cos\phi, tv\sin\phi, t)$ where ϕ is the direction angle of the motion vector which belongs to $[0, 2\pi[$, and t is the time duration of segment. At this level of the analysis the segments are no longer enumerable and we refer to probability density of segment which is

$$p(s_c) = \frac{1}{2\pi}\tau e^{-\tau t}$$

since all angles have same probability.

An emission segment s_e is a space time vector $(r\cos\phi, r\sin\phi, \epsilon)$ with $r \in [0, 1[$.

The quantity ϵ is the time needed for a transmission assumed being small compared to typical node mobility, practically $\epsilon = 0$. W.l.o.g, we take later $\epsilon = 0$ in order to insist on the fact that transmission times are infinitely smaller than moving times. With respect to variable ϕ and r, we have the density

$$p(s_e) = \nu r \ .$$

Notice that a journey of k segments is made of k space vectors and $p(\mathcal{C})$ is therefore a density in vector of dimension $3k$.

2.1.2 Journey Laplace transform

Let ζ be a space vector and θ a scalar. We denote $w(\zeta,\theta)$ the journey Laplace transform defined by

$$\begin{array}{rcl} w(\zeta,\theta) & = & E(\exp(-\zeta.Z(\mathcal{C})-\theta T(\mathcal{C}))) \\ & = & \sum_{\mathcal{C}} p(\mathcal{C})\exp(-\zeta.Z(\mathcal{C})-\theta T(\mathcal{C})) \end{array}$$

defined for a domain definition for (ζ,θ). Notice that $\zeta.Z(\mathcal{C})$ is the dot product of two space vectors.

By virtue of Laplace transform we have

$$p(\mathbf{z},t) = (\frac{1}{2i\pi})^3 \int\int w(\zeta,\theta)e^{\zeta.\mathbf{z}+t\theta}d\zeta\frac{d\theta}{\theta} \tag{1}$$

where the integration domains are plans parallel to the imaginary plan in the definition domain. And in this case the quantity $p(\mathbf{z},t)$ is the average density of journeys that arrive at $\mathbf{z}$ before time t.

A journey being an arbitrary combination of carry segments and emission segments, we have the simple expression inspired from combinatorial analysis:

$$w(\zeta,\theta) = \frac{1}{1-\sum_{s_c} p(s_c)e^{-\langle s_c,(\zeta,\theta)\rangle} - \sum_{s_e} p(s_e)e^{-\langle s_e,(\zeta,\theta)\rangle}} \tag{2}$$

where the quantity $\langle s_c,(\zeta,\theta)\rangle$ is the dot product between two space-time vectors.

We have the expression

$$\begin{array}{rcl} \sum_{s_c} p(s_c)e^{-\langle s_c,(\zeta,\theta)\rangle} & = & \int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^\infty \tau e^{|\zeta|vt\cos\phi}e^{-\theta t}e^{-\tau t}dt \\ & = & \frac{\tau}{\sqrt{(\tau+\theta)^2-|\zeta|^2v^2}} \end{array}$$

Similarly

$$\begin{array}{rcl} \sum_{s_e} p(s_e)e^{-\langle s_e,(\zeta,\theta)\rangle} & = & \int_0^{2\pi}d\phi\int_0^1 \nu r dr e^{r|\zeta|\cos\phi} \\ & = & \pi\nu\Psi(|\zeta|) \end{array}$$

where

$$\Psi(x) = \frac{2}{x}I_1(x) = \sum_{k\geq 0}(\frac{x}{2})^{2k}\frac{1}{(k+1)!k!}.$$

2.2 Saddle point analysis

Equation (1) for $p(\mathbf{z},t)$ can be rewritten as

$$(\frac{1}{2i\pi})^3\int\int w(\zeta_0+i\theta,\theta_0+i\theta)e^{\langle(\zeta_0,\theta_0),(\mathbf{z},t)\rangle+i\langle(\zeta,\theta),(\mathbf{z},t)\rangle}d\zeta\frac{d\theta}{\theta_0+i\theta} \tag{3}$$

where the integration domain are real plans.

The key of the analysis is the set $\mathcal{K}$ of pairs (ρ,θ) such that $D(\rho,\theta)=1$, called the *Kernel*.

Theorem 2 *When $|\mathbf{z}|$ and t tend both to infinity we have*

$$p(\mathbf{z},t) = (1+O(\frac{1}{\sqrt{t}}))\frac{\exp(-\rho_0|\mathbf{z}|+\theta_0 t)}{2\pi\theta_0\sqrt{\frac{D_\theta D_\rho}{\rho_0}\nabla_2 D(t,|\mathbf{z}|)}}$$

where (ρ_0,θ_0) is the element of $\mathcal{K}$ that minimizes $-\rho|\mathbf{z}|+\theta t$. Quantity $D_\rho = \frac{\partial}{\partial\rho}D$, $D_\theta=\frac{\partial}{\partial\theta}D$ and $\nabla_2 D(x,y) = x^2\frac{\partial^2 D}{\partial\rho^2}+y^2\frac{\partial^2 D}{\partial\theta^2}+2xy\frac{\partial^2 D}{\partial\rho\partial\theta}$

Proof The function $1-D(\rho,\theta)$ has a definition domain $\{(\rho,\theta), \Re((\theta+\tau)^2 - \rho^2 v^2) > 0\}$ and, when θ varies, it has a simple root at

$$\theta(\rho) = \sqrt{(\frac{\tau}{1-\pi\nu\Psi(\rho)})^2 + \rho^2 v^2} - \tau.$$

Notice that $(\rho,\theta(\rho))$ describes the set $\mathcal{K}$. In order to have $\rho > 0$ for some elements in $\mathcal{K}$ and to apply a consistent analysis we need the condition $\pi\nu < 1$.

Therefore the residues analysis gives:

$$p(|\mathbf{z}|,t) = I(\mathbf{z},t) + R(\mathbf{z},t)$$

where

$$I(\mathbf{z},t) = \frac{1}{(2i\pi)^2}\int \frac{\exp\langle(\zeta,\theta(|\zeta|)),(\mathbf{z},t)\rangle}{\theta(\rho)D_\theta(\rho,\theta(\rho))} d\zeta$$

with $D_\theta = \frac{\partial}{\partial\rho}D$, and $R(\mathbf{z},t)$ is the integral of $\frac{\exp(\langle(\zeta,\theta),(\mathbf{z},t)\rangle}{(1-D(|\zeta|,\theta)\theta}$ when $\rho v - \tau < \Re(\theta) < \theta(\rho)$.

We will show at the end that $R(\mathbf{z},t) = O(e^{-Bt} I(\mathbf{z},t))$ for some $B > 0$.

We first focus on $I(\mathbf{z},t)$ by using saddle point techniques. Let ζ_0 be the value that minimizes $(\zeta, \mathbf{z} + \theta(|\zeta|)t)$. Obviously $\zeta_0 = -\frac{\rho_0}{|\mathbf{z}|}\mathbf{z}$ with ρ_0 that minimizes $-\rho|\mathbf{z}| + \theta(\rho)t$.

Let θ' and θ'' be the first and second derivatives of $\theta(\rho)$ respectively. We already know that $\theta' = \frac{|\mathbf{z}|}{t}$. Since $D(\rho,\theta(\rho)) = 1$, by derivation with respect to ρ we have $D_\rho + D_\theta\theta' = 0$ and, by second derivation, $\nabla_2 D(1,\theta') + D_\theta\theta'' = 0$ at $\rho = \rho_0$. Without loss of generality we assume that $\mathbf{z} = (-|\mathbf{z}|, 0)$ and $\zeta = (-\rho_0, 0) + (x,y)$, thus

$$|\zeta| = \rho_0 - x + \frac{1}{2\rho_0}y^2 + O(x^3+y^3)$$

and in turn

$$\zeta.\mathbf{z} + \theta(|\zeta|)t = -\rho_0|\mathbf{z}| + \theta_0 t + \frac{t}{2}(\theta'' x^2 + \frac{\theta'}{\rho_0}y^2) + O((x^3+y^3)t)$$

By change of variable $\zeta = (-\rho_0,0) + \frac{i}{\sqrt{t}}(x,y)$ we get

$$\zeta.\mathbf{z} + \theta(|\zeta|)t = -\rho_0|\mathbf{z}| + \theta_0 t - \tfrac{1}{2}(\theta'' x^2 + \tfrac{\theta'}{\rho_0}y^2) + O((x^3+y^3)t^{-1/2}) \quad (4)$$

$$\begin{aligned} I(\mathbf{z},t) &= \frac{\exp(-\rho_0|\mathbf{z}|+\theta_0 t)}{(2\pi)^2}\int\int \frac{\exp(\frac{1}{2}(\theta'' x^2 + \frac{\theta'}{\rho_0}y^2))}{\theta D_\theta} dxdy \times (1+O(t^{-1/2})) \\ &= \frac{\exp(-\rho_0|\mathbf{z}|+\theta_0 t)}{2\pi\theta_0 D_\theta\sqrt{\frac{\theta'\theta''}{\rho_0}}}(1+O(t^{-1/2}))\ . \end{aligned} \quad (5)$$

To terminate with integral $R(\mathbf{z},t)$ we identify

$$B = \theta_0 - \rho_0 v + \tau.$$

2.3 Information propagation speed

Let $q(\mathbf{z}, t)$ be the probability that there exists a journey that arrives at distance less than 1 to $\mathbf{z}$ before time t.

Theorem 3 *We have the upper-bound:*

$$q(\mathbf{z}, t) \leq \int_{|\mathbf{z}-\mathbf{z}'|<1} p(\mathbf{z}', t) d\mathbf{z}' \ .$$

Therefore $q(\mathbf{z}, t) = \Theta(p(\mathbf{z}, t))$ and clearly $q(\mathbf{z}, t)$ vanishes very quickly when t is smaller than the value such that $-\rho_0|\mathbf{z}| + \theta_0 t = 0$, *i.e.* when $\frac{\theta_0}{\rho_0} = \frac{z}{t} = \theta'(\rho_0)$. This ratio gives the upper-bound of the propagation speed. In other word point (ρ_0, θ_0) which achieves the lowest ratio $\frac{\theta}{\rho}$ in the kernel set $\mathcal{K}$.

2.4 The moving destination

In the previous evaluation we assume that the destination does not move during the propagation of the information. Now we consider that the destination can move as the other nodes, starting at position $\mathbf{z}$ at time $t = 0$.

Theorem 4 *When the destination moves as the other nodes in the network then the asymptotic propagation speed upper bound does not change when* $(\mathbf{z}, t)$ *tend to infinity.*

For this end it suffices to multiply the journey Laplace transform $w(\zeta, \theta)$ with the Laplace transform of the node excursion from its original position. The excursion Laplace transform is obtained from carry segment Laplace transform and has expression $\frac{1}{f(|\zeta|,\rho)-\tau}$ with $f(\rho, \theta) = \sqrt{(\theta + \tau)^2 - \rho^2 v^2}$. The new Laplace transform has two sets of poles, the set $\mathcal{K}$ and the set $\mathcal{K}_2$ corresponding to the set $\{(\rho, \theta), \theta = \rho v - \tau\}$. The last set is dominated on the right by CK, for all $(\rho, \theta_2) \in \mathcal{K}_2$, there is a (ρ, θ) in $\mathcal{K}$ with $\theta > \theta_2 + B$. Therefore the contributions from $\mathcal{K}_2$ will be exponentially negligible (of order $\exp(-Bt)$) with regard to the main contribution from $\mathcal{K}$.

The main contribution from $\mathcal{K}$ gives

$$p(\mathbf{z}, t) \sim \frac{\exp(-\rho_0|\mathbf{z}| + \theta_0 t)}{(f(\rho_0, \theta_0) - \tau) 2\pi\theta_0 \sqrt{\frac{D_\theta D_\rho}{\rho_0} \nabla_2 D(t, |\mathbf{z}|)}}$$

and the propagation speed upper-bound does not change from the value computed in the last subsection.

3 Realistic Approach (Proof of Theorem 1)

In this approach we assume that the nodes can transmit only when they meet new neighbors or when they receive the beacon. According to [8] when nodes move at speed v with isotropic direction, then the frequency f at which new neighbors appear satisfies $f = \frac{8v}{\pi}\nu$.

Lemma 1 *The journey Laplace transform with realistic approach has the expression*

$$w(\zeta,\theta)=\frac{R(|\zeta|,\theta)}{1-D(|\zeta|,\theta)}$$

with

$$D(\rho,\theta)=\frac{1}{\sqrt{(\theta+\tau)^2-\rho^2v^2}}\left(\tau+\frac{fI_0(\rho)}{1-\pi\nu\Psi(\rho)}\right)$$

where

$$I_0(x)=\sum_{k\geq 0}(\frac{x}{2})^{2k}\frac{1}{(k!)^2}$$

and

$$R(\rho,\theta)=\frac{1}{\sqrt{(\theta+\tau)^2-\rho^2v^2}}.$$

Proof A typical journey is an arbitrary mixture of carry segments s_c and carry-and-transmit segments s_{ce}. A carry-and-transmit segment is made of a carry-to-neighbor segment s_{cn} and transmit-to-neighbor segment s_{tn} followed by an arbitrary number of emission segments s_e. The generating function of the arbitrary number of emission segment is $\frac{1}{1-\pi\nu\Psi(\rho)}$.

The carry-to-neighbor segment is a space-time vector $(v\cos\phi, v\sin\phi, t)$ where ϕ is a number in $[0,2\pi[$ and t is a non negative number. We have the density

$$p(s_{cn})=e^{-\tau t}\frac{f}{2\pi}.$$

The generating function:

$$E(e^{\langle(\zeta,\theta),s_{cn}\rangle})=\frac{f}{\sqrt{(\rho+\tau)^2-\rho^2v^2}}$$

For the transmit-to-neighbor segment we assume that the beacon is transmitted to the new neighbor only, at distance 1, the other neighbors having already received the beacon. Therefore a transmit-to-neighbor segment is a space-time vector $(\cos\phi,\sin\phi,0)$, and $p(s_{tn})=\frac{1}{2\pi}$.

Computing the generating function yields.

$$\begin{array}{rcl}E(e^{\langle(\zeta,\theta),s_{tn}\rangle}) & = & \int_0^{2\pi}e^{|\zeta|\cos\phi}\frac{d\phi}{2\pi}\\ & = & I_0(|\zeta|)\end{array}$$

Finally the generating function of the carry-and-transmit segment is

$$\frac{fI_0(\rho)}{\sqrt{(\theta+\tau)^2-\rho^2v^2(1-\pi\nu\Psi(\rho))}}\ .$$

Therefore the function $w(\zeta,\theta)$ is in denominator $1-E(e^{\langle(\zeta,\theta),s_c\rangle}-E(e^{\langle(\zeta,\theta),s_{ce}\rangle}$ and in numerator, the function $\frac{1}{\sqrt{(\rho+\tau)^2-\rho^2v^2}}$, the excursion Laplace transform, to indicate that the last straight line before the destination. This terminates the proof.

Using the methodology developed in the simplified approach section, we now prove Theorem 1 as follows.

Theorem 1 An upper bound of the information propagation speed in a network (where node radio range is R, node density is ν, node speed is v, node direction change rate is τ) is the smallest ratio $\frac{\theta}{\rho}$ attained by the elements (ρ,θ) of the set $\mathcal{K}$ made of the non negative tuples (ρ,θ) that are root of

$$\sqrt{(\tau+\theta)^2-\rho^2v^2}-\tau-\frac{\frac{8vR}{\pi}\nu I_0(\rho R)}{1-\pi R^2\nu\Psi(\rho R)}.$$

Proof The Kernel of $w(\zeta,\theta)$ is the root of the denominator $\sqrt{(\tau+\theta)^2-\rho^2v^2}-\tau-\frac{\frac{8vR}{\pi}\nu I_0(\rho R)}{1-\pi R^2\nu\Psi(\rho R)}$. Therefore, following the asymptotic analysis of the average number of journeys, the propagation speed upperbound is given by the minimum ratio $\frac{\theta}{\rho}$ of $(\rho,\theta)\in\mathcal{K}$.

Corollary 1 Let $v>0$ and $\tau>0$, when $\nu\to 0$, the propagation speed is asymptotically equivalent to $4v\sqrt{\frac{\nu vR}{\tau\pi}}$.

Proof Let (ρ,θ) be an element of the set $\mathcal{K}$. W.l.o.g. for $R=1$, we have $\theta=\sqrt{(\tau+\nu H(\rho))^2+\rho^2v^2}-\tau$, with $H(\rho)=\frac{\frac{8}{\pi}I_0(\rho)}{1-\pi\nu\Psi(\rho)}$. We have

$$H(\rho)=\frac{8v}{(1-\pi\nu)\pi}+O(\rho^2).$$

Therefore

$$\theta=\sqrt{\tau^2+\rho^2v^2}-\tau+\frac{\tau}{\sqrt{\tau^2+\rho^2v^2}}H(\rho)\nu+O(\nu^2).$$

The ratio

$$\begin{aligned}\frac{\theta}{\rho} &= \frac{H(\rho)\nu}{\rho}\frac{\tau}{\tau^2+\rho^2v^2}+\frac{\sqrt{\tau^2+\rho^2v^2}-\tau}{\rho}+O(\frac{\nu^2}{\rho})\\ &= \frac{H(0)\nu}{\rho}+\frac{\rho v^2}{2\tau}+O(\frac{\nu^2}{\rho}+\nu\rho^2)\end{aligned}$$

Quantity $\frac{H(0)\nu}{\rho}+\frac{\rho v^2}{2\tau}$ is minimized with value $v\sqrt{\frac{2\nu H(0)}{\tau}}$ attained at $\rho=\frac{\sqrt{2\nu\tau H(0)}}{v}$. Therefore $\frac{\theta}{\rho}$ is minimized at value $v\sqrt{\frac{2\nu H(0)}{\tau}}+O(\nu^{3/2})$.

3.1 The random way-point limit

The random way-point model is equivalent to set $\tau=0$. It is not really the random way-point model, since the true random way-point model assumes a finite convex map and our model is developed on infinite maps. Indeed the node density in the finite random way-point is not uniform and there is no independence between speed angle and node position. Anyhow if we consider a disk map that grows to infinity, then we will get our infinite random way-point model. There is nothing particular about our random way-point model, excepted that the Kernel set $\mathcal{K}$ is made of points $(\rho,\theta(\rho))$ where $\theta(\rho)=v\sqrt{\rho^2+G(\rho)^2}$ with $G(\rho)=\frac{\frac{8\nu}{\pi}I_0(\rho)}{1-\pi\nu\Psi(\rho)}$. In this case the upper-bound speed is proportional to v with factor of proportionality equal to $\sqrt{\rho_0^2+G(\rho_0)^2}$ where ρ_0 minimizes $\frac{G(\rho)}{\rho}$.

Since $\frac{G(\rho_0)}{\rho_0}=\frac{8\nu}{\pi}\min\frac{I_0(\rho)}{\rho}+O(\nu^2)$ we get the estimate $(1+O(\nu^2))v$, proving Corollary 2.

3.2 The Brownian limit

The Brownian limit is more interesting. It happens when $\tau \to \infty$. But in this case if the speed remains constant, then at the limit the nodes does not move anymore. Therefore we have to accelerate the speed v suitably to get significant results. This is done by increasing the speed v such that $\frac{v^2}{\tau}$ tends to a limit σ called the Brownian variance spread factor.

In this case we get

$$\sqrt{(\tau+\theta)^2 - \rho^2 v^2} - \tau = \theta - \frac{v^2}{\tau}\rho^2 + O(\frac{1}{\tau^2}).$$

This leads to

$$\theta(\rho) = vG(\rho) + \sigma\rho^2 + O(\frac{1}{\tau^2}).$$

If $\sigma\rho^2$ is negligible in front of $vG(\rho)$ then we get the same estimate than with the random way point model. This estimate is clearly too large, because at fixed speed v, the nodes do not really move when $\tau \to \infty$. The reason for this discrepancy is that when a node enters a disk unit of another node then by virtue of the random walk it comes out and in again frequently, and this artificially adds to the neighbor creation rate f (this is the same neighbor that shows up and shows down). This particular model needs to be refined.

4 Bound values and Simulations

In this section, we first present some calculations to illustrate the behavior of our upper bound on the information propagation speed, then assess the accuracy of our theoretical upper bound by comparing it with simulation results.

4.1 Slowness of Information Propagation.

To illustrate the behavior of the upper bound on the information propagation speed when the mobile density ν varies, we define the *slowness*, *i.e.,* the inverse of the information propagation speed, for which our theroretical study now provides lower bounds.

For the calculations, we use a Unit-Disk Graph model, where mobiles speed is $1s^{-1}$. Two commonly used mobility models are simulated: Random Way-Point model (which corresponds to our setting $\tau = 0$ as described in subsection 3.1) and Random Walk model (which corresponds to our setting $\tau = 0.1$). Plotting results (obtained by numerical resolution) of our theoretical lower bound are presented in Figure 1 and in Figure 2 respectively.

We remark that the slowness drops to 0 at $\nu = 1/\pi$ which corresponds to the limit of our model. Recall, that this is a lower bound of slowness (equivalent to the upper bound on the propagation speed). Actual slowness should continue to be non-zero beyond $\nu = 1/\pi$.

For the Random Way Point (*i.e.,* $\tau = 0$), we notice that the slowness is in $1 - O(\nu^2)$ confirming the Corollary 2. For the Random Walk, we notice that the slowness is unbounded when $\nu \to 0$ confirming the $O(\frac{1}{\sqrt{\nu}})$ theoretical behavior proved in Corollary 1.

4.2 Accuracy: Theoretical Bounds vs. Simulations.

We now evaluate the accuracy of our theoretical upper bound in different scenarios by comparing it to the average information propagation time obtained by simulating a full broadcast (as described in Section 1). In all cases (see Figure 3 to Figure 10), we provide the simulated average information propagation time versus the distance (plots), and display the theoretical bound slope (in color). For the theoretical bound slope, the starting point is placed manually to best-fit the constant values which are not calculated in our model. Therefore the analysis of position of the slope as an upper or lower bound is irrelevant. Again, what is important is the comparison of the two slopes at infinity.

Again, we study the same two popular mobility models: Random Way-Point (*i.e.,* $\tau = 0$) and Random Walk (*i.e.,* $\tau = 0.1$) for different density and area values ($\nu = 0.025$ on a 80×80 square, $\nu = 0.05$ on a 60×60 square, $\nu = 0.1$ on a 40×40 square, $\nu = 0.2$ on a 30×30 square, respectively). For all the simulations, we use a Unit-Disk Graph model, where mobiles speed is $1s^{-1}$.

For both mobility models, the simulations show that the theoretical bound provides an accurate slope until the network is too dense ($\nu = 0.2$ on a 30×30 square), but the slope is still a lower bound as proved in the Theorems.

5 Conclusion

In this paper we have initiated a partial characterisation of the information propagation speed of Delay/Disruption Tolerant mobile Networks (DTNs) and Intermittently Connected mobile Networks (ICNs) by providing a theoretical upper bound. The model used in our analytical study is sufficiently general to encapsulate many popular mobility models (such as Random Way-Point and Random Walk). Simulations and calculations for several scenarios show the accuracy of our upper bound, specifically when the networks are sparse and may never be fully connected. Future investigations should consider extending the analysis to larger dimensions and neighboring models different from Unit-Disk Graphs (*e.g.,* quasi-disk graphs, probabilistic models), as well as designing an accurate lower bound for the information propagation speed.

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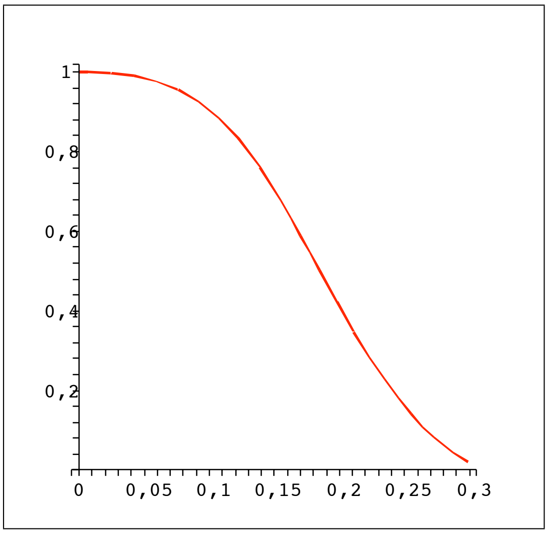

Figure 1: Theoretical lower bound of slowness versus mobile density ν when $\tau = 0$

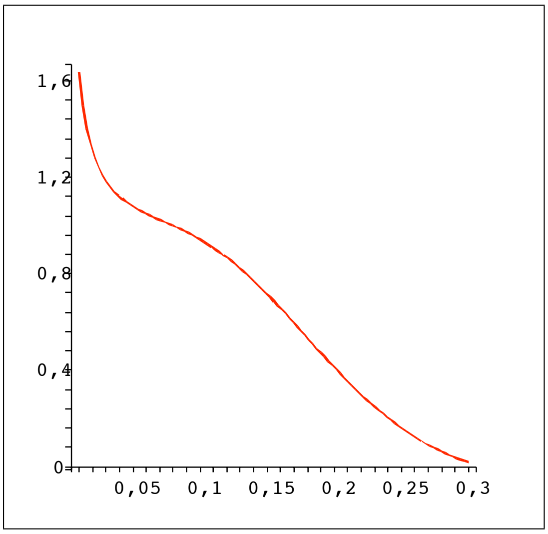

Figure 2: Theoretical lower bound of slowness versus mobile density ν when $\tau = 0.1$

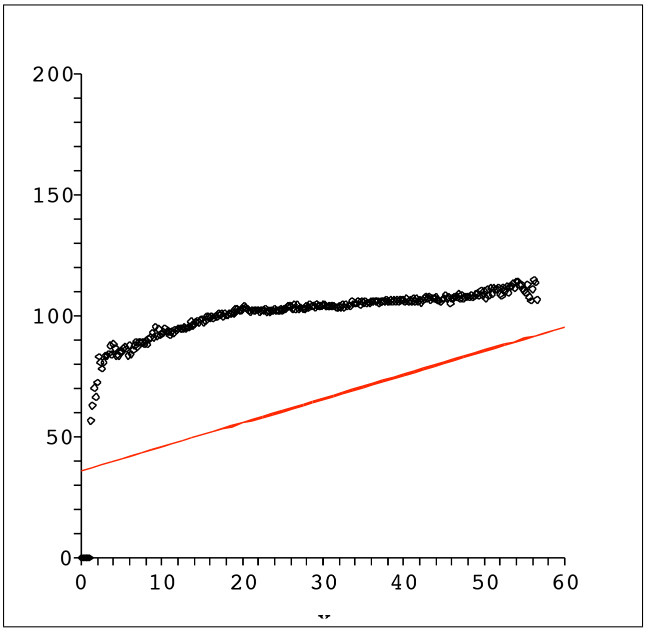

Figure 3: Average propagation time versus distance to source compared with theoretical slope, for $\tau = 0$ and $\nu = 0.025$, simulated on a 80×80 square

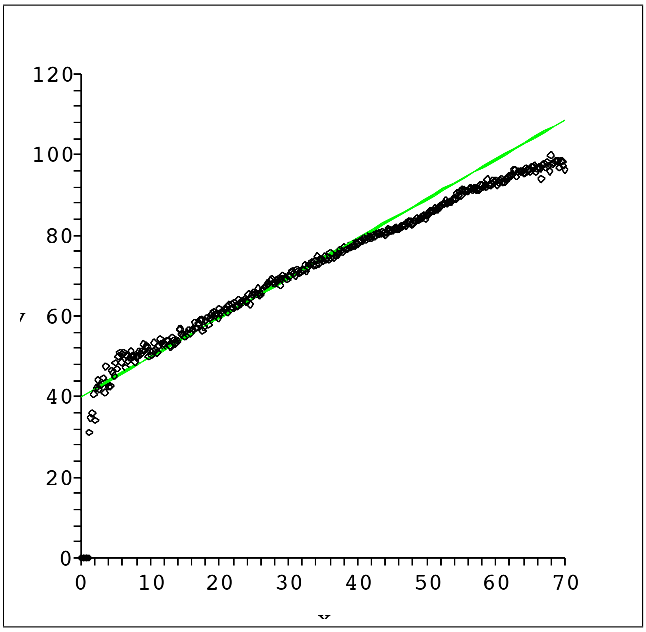

Figure 4: Average propagation time versus distance to source compared with theoretical slope, for $\tau = 0$ and $\nu = 0.05$, simulated on a 50×50 square

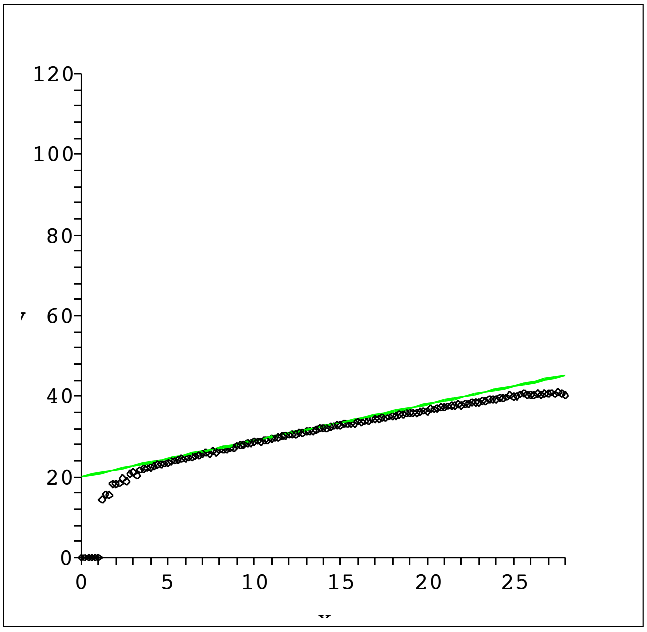

Figure 5: Average propagation time versus distance to source compared with theoretical slope, for $\tau = 0$ and $\nu = 0.1$, simulated on a 40×40 square

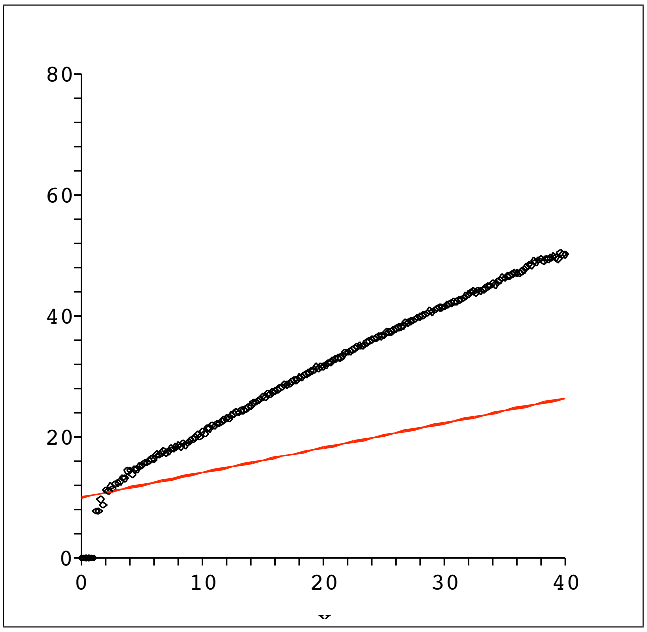

Figure 6: Average propagation time versus distance to source compared with theoretical slope, for $\tau = 0$ and $\nu = 0.2$, simulated on a 30×30 square

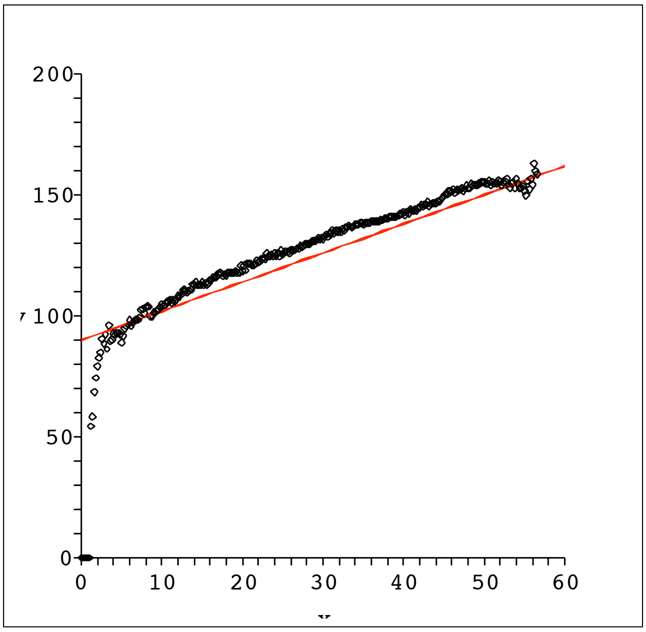

Figure 7: Average propagation time versus distance to source compared with theoretical slope, for $\tau = 0.1$ and $\nu = 0.025$, simulated on a 80×80 square

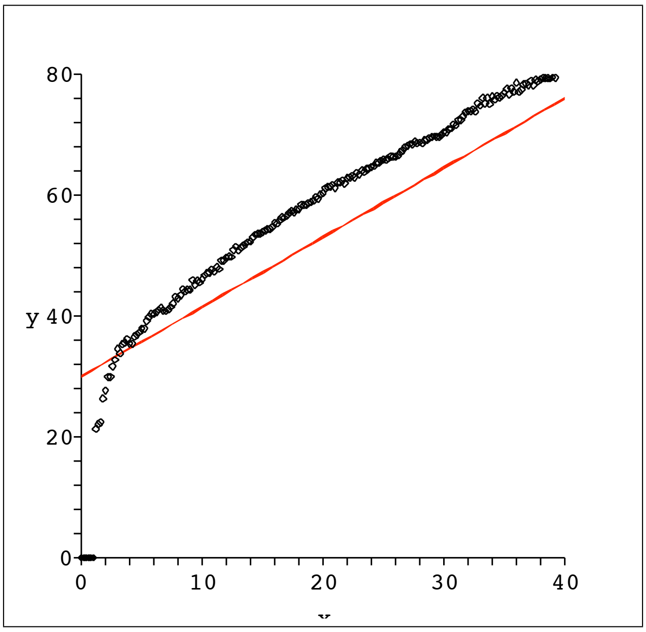

Figure 8: Average propagation time versus distance to source compared with theoretical slope, for $\tau = 0.1$ and $\nu = 0.05$, simulated on a 60×60 square

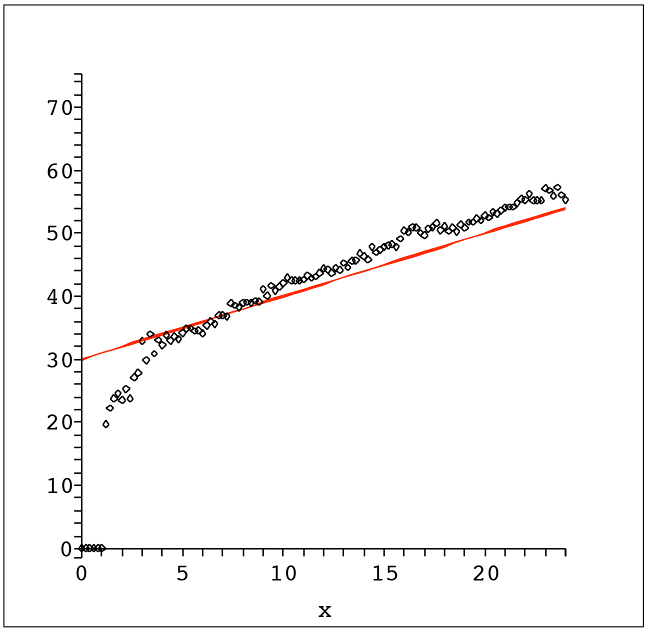

Figure 9: Average propagation time versus distance to source compared with theoretical slope, for $\tau = 0.1$ and $\nu = 0.1$, simulated on a 40×40 square

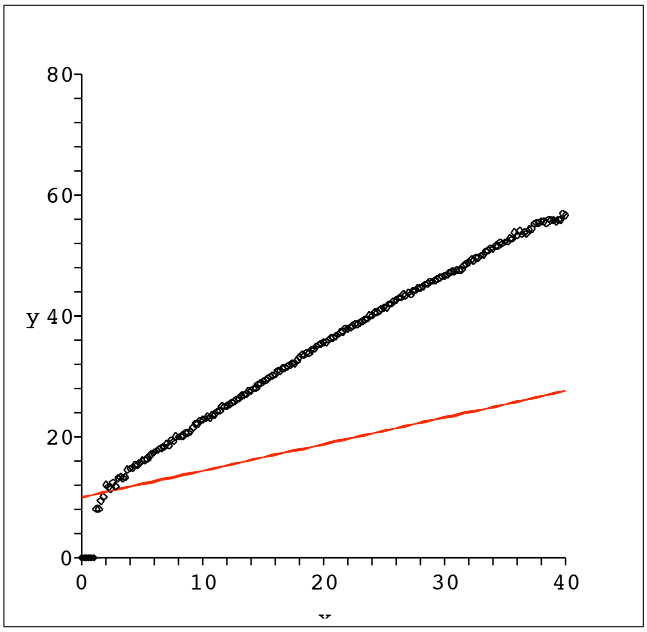

Figure 10: Average propagation time versus distance to source compared with theoretical slope, for $\tau = 0.1$ and $\nu = 0.2$, simulated on a 30×30 square

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